

Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 1 (Individual)

香港数学竞赛 (2006 – 2007)

决赛项目 1 (个人)

除非特别声明，答案须用数字表达，并化至最简。

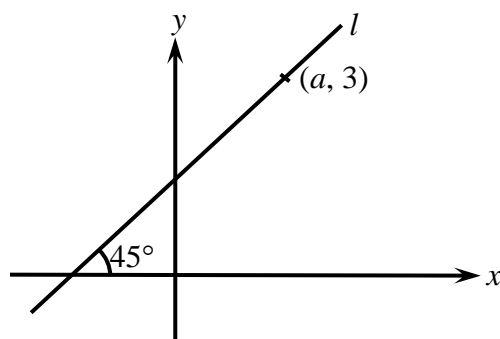
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 设 $\sqrt{a} = \sqrt{7+\sqrt{13}} - \sqrt{7-\sqrt{13}}$ ，求 a 的值。

Let $\sqrt{a} = \sqrt{7+\sqrt{13}} - \sqrt{7-\sqrt{13}}$, find the value of a .

2. 如图一，直线 l 经过点 $(a, 3)$ 并与 x -轴成 45° 夹角。若 l 的方程是 $x + my + n = 0$ 及 $b = |1 + m + n|$ ，求 b 的值。

In Figure 1, the straight line l passes through the point $(a, 3)$ and makes an angle 45° with the x -axis. If the equation of l is $x + my + n = 0$ and $b = |1 + m + n|$, find the value of b .



图一

Figure 1

3. 若 $x - b$ 为 $x^3 - 6x^2 + 11x + c$ 的因式，求 c 的值。

If $x - b$ is a factor of $x^3 - 6x^2 + 11x + c$, find the value of c .

4. 若 $\cos x + \sin x = -\frac{c}{5}$ 及 $d = \tan x + \cot x$, 求 d 的值。

If $\cos x + \sin x = -\frac{c}{5}$ and $d = \tan x + \cot x$, find the value of d .



Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 2 (Individual)

香港数学竞赛 (2006 – 2007)

决赛项目 2 (个人)

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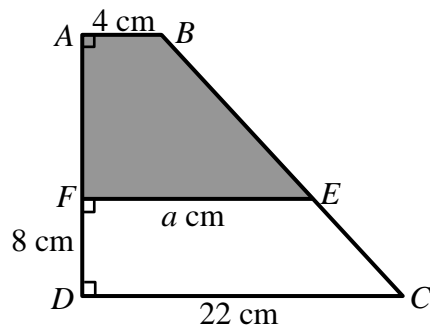
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 设 $n = 1 + 3 + 5 + \cdots + 31$ 及 $m = 2 + 4 + 6 + \cdots + 32$ 。若 $a = m - n$ ，求 a 的值。

Let $n = 1 + 3 + 5 + \cdots + 31$ and $m = 2 + 4 + 6 + \cdots + 32$. If $a = m - n$, find the value of a .

2. 如图一， $ABCD$ 是一梯形， $AB = 4\text{cm}$ 、 $EF = a\text{cm}$ 、 $CD = 22\text{cm}$ 及 $FD = 8\text{cm}$ 。若 $ABEF$ 的面积是 $b\text{cm}^2$ ，求 b 的值。

In Figure 1, $ABCD$ is a trapezium, $AB = 4\text{cm}$, $EF = a\text{cm}$, $CD = 22\text{cm}$ and $FD = 8\text{cm}$. If the area of $ABEF$ is $b\text{cm}^2$, find the value of b .

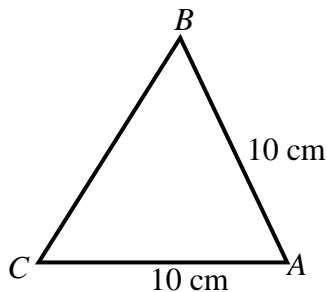


图一

Figure 1

3. 如图二， $\triangle ABC$ 是一个三角形， $AB = AC = 10$ cm 及 $\angle ABC = b^\circ - 100^\circ$ 。若 $\triangle ABC$ 有 c 条对称轴，求 c 的值。

In Figure 2, $\triangle ABC$ is a triangle, $AB = AC = 10$ cm and $\angle ABC = b^\circ - 100^\circ$. If $\triangle ABC$ has c axis of symmetry, find the value of c .



图二
Figure 2

4. 设 d 为方程 $cx^{\frac{2}{3}} - 8x^{\frac{1}{3}} + 4 = 0$ 的最小实根，求 d 的值。

Let d be the least real root of the equation $cx^{\frac{2}{3}} - 8x^{\frac{1}{3}} + 4 = 0$, find the value of d .

Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 3 (Individual)

香港数学竞赛 (2006 – 2007)

决赛项目 3 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 设 $a = \cos^4 \theta - \sin^4 \theta - 2\cos^2 \theta$ ，求 a 的值。

Suppose that $a = \cos^4 \theta - \sin^4 \theta - 2\cos^2 \theta$, find the value of a .

2. 若 $x^y = 3$ 及 $b = x^{3y} + 10a$ ，求 b 的值。

If $x^y = 3$ and $b = x^{3y} + 10a$, find the value of b .

3. 若有 c 个正整数 n 使得 $\frac{n+b}{n-7}$ 也是正整数，求 c 的值。

If there is(are) c positive integer(s) n such that $\frac{n+b}{n-7}$ is also a positive integer, find the value of c .

4. 设 $d = \log_4 2 + \log_4 4 + \log_4 8 + \cdots + \log_4 2^c$ ，求 d 的值。

Suppose that $d = \log_4 2 + \log_4 4 + \log_4 8 + \cdots + \log_4 2^c$, find the value of d .

Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 4 (Individual)

香港数学竞赛 (2006 – 2007)

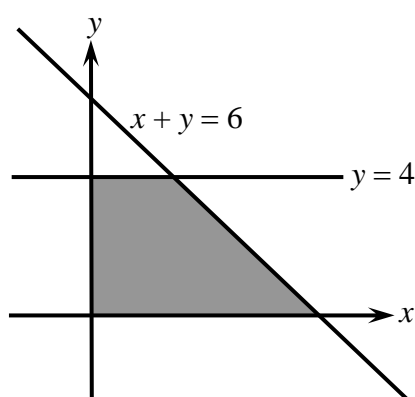
决赛项目 4 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如图一，设直线 $x + y = 6$ 、 $y = 4$ 、 $x = 0$ 及 $y = 0$ 所围成的封闭区域的面积是 A 平方单位，求 A 的值。

In Figure 1, let the area of the closed region bounded by the straight lines $x + y = 6$, $y = 4$, $x = 0$ and $y = 0$ be A square units, find the value of A .



图一

Figure 1

2. 设 $[x]$ 表示不大于 x 的最大整数，例如 $[2.5] = 2$ 。若 b 满足方程组
$$\begin{cases} Ax^2 - 4 = 0 \\ 3 + 2(x + [x]) = 0 \end{cases}$$
，求 b 的值。

Let $[x]$ be the largest integer that is not greater than x , for example, $[2.5] = 2$. If b satisfies the

system of equations
$$\begin{cases} Ax^2 - 4 = 0 \\ 3 + 2(x + [x]) = 0 \end{cases}$$
, find the value of b .

3. 设 c 为 $\left(2x + \frac{b}{\sqrt{x}}\right)^3$ 展开式中的常数项, 求 c 的值。

Let c be the constant term in the expansion of $\left(2x + \frac{b}{\sqrt{x}}\right)^3$, find the value of c .

4. 若满足不等式 $\left|\frac{x}{2} - \sqrt{2}\right| < c$ 的整数解有 d 个, 求 d 的值。

If the number of integral solutions of the inequality $\left|\frac{x}{2} - \sqrt{2}\right| < c$ is d , find the value of d .